On the Observation of Decoherence with a Movable Mirror

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Recently it has been proposed to use parity as a measure of the mechanism behind decoherence or the transformation from quantum to classical. Here, we show that the proposed experiment is more feasible than previously thought, as even an initial thermal state would exhibit the hypothesized symmetry breaking. – Pacs: 03.75.Be, 03.65.Nk

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"The ball is in box 1 *and* the ball is in box 2" versus "the ball is in box 1 *or* the ball is in box 2", is well known to describe the difference between the quantum world and our classical reality. The transition from a position superposition to a well localized state is clearly a spatial symmetry breaking transition (let the symmetry axis lie half way between the boxes and perpendicular to the line that joins them). One should note that although a *symmetric* density matrix in position space ρ_f is formed in the place of that describing the initial superposition ρ_i, symmetry has been broken, as may be revealed by the different outcomes of applying the parity operator $\hat{P}$ onto the above two density matrices, e. g. in a spatial two dimensional space of $|R\rangle$ and $|L\rangle$

$$\rho_i = \frac{1}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix} \quad \rho_f = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

where

$$\hat{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

and where $\langle \hat{P}_f \rangle = \mathrm{Tr}(\hat{P}\rho_f)/\mathrm{Tr}(\rho_f) = 0$ while $\langle \hat{P}_i \rangle = \pm 1$.

The above symmetry breaking has in the past received many different names, examples being 'an emerging element of reality' or 'localization' and more recently 'dephasing' and 'decoherence'. In attempting to quantify this process of transition into what is sometimes referred to as 'classical reality' or a 'classical state', perhaps the notion of a statistical mixture is the more convenient description, where the declining magnitudes of the off diagonal elements of the density matrix are the measure of the decoherence. There has been much debate on what is the origin of this process and whether or not it is independent of us and our measurements [1]. In order to resolve this issue one has to measure the process.

However, until now the question remains, how can one observe this process in x space without initiating it? Let us elaborate: Observing a quantum system needs coupling to an external detection system. In general, this influences the dynamics and causes symmetry breaking. Hence it is clear that by observing a quantum system, we couple our detectors to it and consequently allow its symmetry to break and thus mask the very process we wish to observe, namely, the emmergence of a statistical mixture independent of our measurement. This may be formally described as

$$P^2 + W^2 = 1, \qquad (1)$$

where P is a particle measure, W a wave measure which is simply the visibility of an interference pattern, and they both take values between 0 and 1 [2].

The particle measure is usually just the ability to predict which path a particle will take, i. e. to predict the outcome of a "box 1 *or* box 2" experiment. However, it may also be referred to as the knowledge K possibly obtained in a which path information measurement [3], and the above equality is then re-written as

$$K^2 + V^2 = 1, \qquad (2)$$

where V is now the visibility.

The answer proposed by Folman et al. [4] to this experimental dilemma is that one searches for symmetry breaking through parity eigen state measurements which do not reveal which path information, i. e. the coupling to the measurement apparatus is facilitated through a parity conserving pointer basis set. Namely, that another measurement basis exists with which the emmergence of a 'classical state' in the form of a statistical mixture can be observed, which is different from the usual 'which path' basis which is simply the previously introduced $|R\rangle$ and $|L\rangle$. This basis, the parity basis, will not, as will be seen in the following, couple to the system in a way that would mask other processes causing spatial decoherence.

It is perhaps interesting to note that relative to the usual basis, the parity basis is not intuitive as a measure of a 'classical state' in the sense that it does not observe 'particles' – the intuitive core of a 'classical state'. Hence, people have usually considered that measuring the appearance of a 'classical state' and looking for 'particles' are one of the same. As the notion of 'particles' is only valid in the 'which path' basis, the latter is the basis commonly used, as is also evident from (1) and (2).

Consequently, re-defining the 'particle measure' P as a 'classical state' measure, i. e. the measure of a statistical mixture and not more, we find that we can be sensitive to P while not suppressing the value of W which is the measure of the quantum state. We should note, that according to the usual definition of P, the above equalities remain of course unbroken also in our case because the 'which path' predictability / knowledge remains zero. In the sense that a 'particle' is just a euphemism for 'classical state' or statistical mixture, we feel that the mathematical definition
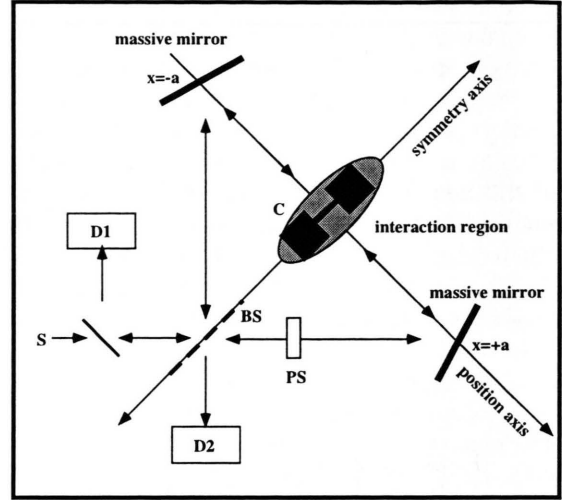

Fig. 1. The open loop interferometer. Here, we position a two sided foil mirror, held in an harmonic potential, into the interaction region C, where the split incoming photon wave function "hits" the foil from both sides. The beam splitter (BS) and the phase shifter (PS) (with a phase shift equal to the phase difference between the reflected and transmitted wave at the beam splitter), form a preparation system which ensures that the photon wave function is symmetric with respect to the symmetry axis. The same system acts as a detection system with parity eigen states. Taking the mirror to be part of the set up, one may say that we entangle the transversing photon to the state of the set-up, which is also in a well defined quantum state with a well defined spatial uncertainty. Using Parity eigen-states to measure the state of the outgoing photon, we learn about the decoherence of the set-up, without affecting it. Thus, we are able to examine the quantum evolution of a macroscopic system.

originally given to P does not encompass fully the stated function of a 'particle measure'.

To give a simple example of the procedure proposed in [4], it is well known that once a chiral molecule has been localized and hence created an element of reality in the form of its well defined handedness, it is no longer in the parity eigen state, displaying once again the symmetry border between classical and quantum. As measuring parity does not localize the molecule, observing a parity change of an isolated molecule could only mean that "something else" localized it. Indeed, Folman et al. have shown how parity eigen state measurements could be used to investigate different models of decoherence. On the other hand, if one tries to observe the molecule's evolution from a parity eigen state into a well defined

handedness by measuring its handedness, they will initiate that evolution themselves.

In this paper we would like to extend the scope of the previously mentioned experimental proposal by showing that even if the initial quantum system is in a thermal state, the expected symmetry breaking signal will still be observable. This has an important consequence as extreme cooling of macroscopic objects to their ground state, or difficult verification of an initial pure state, will not be needed. Consequently, the proposed experiment is much more feasible than previously thought.

Before beginning, let us briefly remind the reader of the experimental set up proposed [4]. We present it in Figure 1: A closed loop triangle interferometer consisting of one beam splitter rather than the usual Mach-Zehnder double beam splitter set up, is turned into an open interferometer (two separate non overlaping optical paths) by introducing a double sided mirror into the set up. The mirror plane coincides with the plane of the beam splitter, which constitutes the spatial symmetry axis of the set up. The beam splitter, including a phase shifter which compensates for the phase difference between the transmitted and reflected beams, prepares an initial symmetric photon. Together with the two detectors, it also acts as a measurement with parity eigen states on the outgoing photon.

After the incoming photon interacts with the mirror, it may excite it or not, where the chance of the latter occurrence is just the Debye-Waller factor $P_{0\rightarrow0}$. However, as there are no symmetry breaking terms in the quantum evolution, the initial symmetry of the system, being composed of a symmetric photon and a symmetric mirror state $\Omega_i = \Phi_i \otimes \Psi_i$, must be conserved. Hence one may write the final wave function of the system as

$$\Omega_f = \sum \Phi_s \otimes \Psi_s + \Phi_{as} \otimes \Psi_{as}, \qquad (3)$$

where 's' and 'as' stand for symmetric and anti-symmetric, and the summation is over all possible foil states.

Indeed, as the parity measurement is in the above basis, it will in fact collapse the mirror, which is entangled to the photon, into a specific symmetry. In addition, measuring the energy of the outgoing photon would collapse the mirror into *one* specific energy eigen state of the harmonic oscillator. We post select into our data set only photons for which the energy

difference ΔE between the incoming and outgoing photon obeys $\Delta E = 2m\hbar\omega$, where m is a natural integer. According to standard quantum evolution, there should be no parity -1 (i. e. anti symmetric) detector clicks in our data set, as the latter are, in the absence of localization, correlated to a foil excitation of an odd number of levels. Namely, within our post selected data set, detector D2 should remain dark. Consequently, any such events in our data set must be a result of a non-standard evolution of our foil through coupling to the environment or through non quantum decoherence processes such as the GRW mechanism [5]. We end here our brief summary of the proposed experiment, where a clear shortcoming of the original discussion was that of being limited to initial pure states, which are hard to achieve experimentally for macroscopic objects.

Let us now start with an initial thermal state. Following the object discussed in the original proposal, we begin by describing the initial density matrix of a foil mirror oscillating in an harmonic potential. Our basis is that of the energy eigen states, where for convenience we limit the infinite basis to N states where $N\hbar\omega \gg k_B T$ and where T is the initial temperature of the mirror.

$$\rho(0) = \begin{pmatrix} p_{N-1} & 0 & . & . & . & 0 & 0 \\ 0 & p_{N-2} & & & & & \\ . & & . & & & & \\ . & & & . & & & \\ . & & & & . & & \\ 0 & & & & & p_1 & 0 \\ 0 & & & & & 0 & p_0 \end{pmatrix},$$

where p_n is simply the Boltzmann population $e^{-(\frac{1}{2}+n)\hbar\omega/k_B T}$. As $\rho(t)$ can always be looked upon as the sum of the density matrices of each possible state i.e. $\rho(t) = \sum l_k \rho_k(t)$ where l_k is determined by $\rho(0) = \sum l_k \rho_k(0)$ and is simply the above Boltzmann factor - we examine in the following the evolution of one specific pure state, and will afterwards sum. Namely, we will proceed along the following path:

$$\langle \hat{P}_p \rangle = \frac{\text{Tr}(\hat{P}\rho_{red})}{\text{Tr}(\rho_{red})} = \frac{\text{Tr}(\hat{P}(\text{Tr}_m\rho))}{\text{Tr}(\text{Tr}_m\rho)} \qquad (4)$$

$$= \frac{\text{Tr}(\hat{P}(\text{Tr}_m\sum l_k\rho_k))}{\text{Tr}(\text{Tr}_m\sum l_k\rho_k)} = \frac{\sum l_k(\text{Tr}(\hat{P}(\text{Tr}_m\rho_k)))}{\sum l_k\text{Tr}(\text{Tr}_m\rho_k)},$$

where $\hat{P}_p$ is the parity measurement on the outgoing photon, ρ_{red} is the reduced density matrix after a partial trace Tr_m over the mirror degrees of freedom, and

P in the energy basis of the outgoing photon is (see Appendix A)

$$
\hat{P} = \begin{pmatrix} 1 & 0 & . & . & & . & 0 & 0 \\ 0 & -1 & & & & & & \\ . & & . & & & & & \\ . & & & (-1)^n & & & & \\ . & & & & . & & & \\ 0 & & & & & -1 & 0 \\ 0 & & & & & 0 & 1 \end{pmatrix}.
$$

The latter basis is $|j = -N\rangle, |-N+1\rangle, ..., |N-1\rangle, |N\rangle$, where the state $|j\rangle$ means to how many harmonic oscillator levels the mirror has jumped due to the interaction with the photon. Note that de-excitations of the mirror are in principle also valid. Obviously for an even j, the above parity operator acting on the $|j\rangle\langle j|$ matrix element would return a positive number. In the context of symmetric $|S\rangle$ and anti-symmetric $|AS\rangle$ outgoing photons, and taking N to be even, the above basis can be written as $|S_{-N}\rangle, |AS_{-N+1}\rangle, ..., |AS_{N-1}\rangle, |S_N\rangle$.

It was shown in the original proposal that for any initial mirror state $|n\rangle$ the final state of the mirror-photon system is

$$
|\Omega_f^n\rangle = 2\cos(k^\Delta x)|n\rangle_m \frac{1}{\sqrt{2}}(|1\rangle|0\rangle + |0\rangle|1\rangle)_p \quad (5)
$$

$$
+ 2i\sin(k^\Delta x)|n\rangle_m \frac{1}{\sqrt{2}}(|1\rangle|0\rangle - |0\rangle|1\rangle)_p,
$$

where k^Δ is simply the vectorial sum $k^\Delta = k1 + k2$, where $k1$ and $k2$ are the momenta of the incoming and outgoing photon respectively.

As our energy post selection does not allow for odd excitations, the remaining final state in our data set is

$$
|\Omega_f^n\rangle = 2\cos(k^\Delta x)|n\rangle_m \frac{1}{\sqrt{2}}(|1\rangle|0\rangle + |0\rangle|1\rangle)_p \quad (6)
$$

$$
= \alpha_0|0\rangle_m|S_{-n}\rangle_p + \alpha_2|2\rangle_m|S_{-n+2}\rangle_p + ...
$$

Indeed, it is already clear at this stage that for any given initial pure state our data set will only include parity +1 photons, which means our apparatus will be sensitive to the symmetry breaking expected to appear in the form of anti symmetric photons. A summation over many such initial states that form the initial thermal state would not change this situation. However, let us arrive at this conclusion in a formal way by tracing over the degrees of freedom of the mirror to find
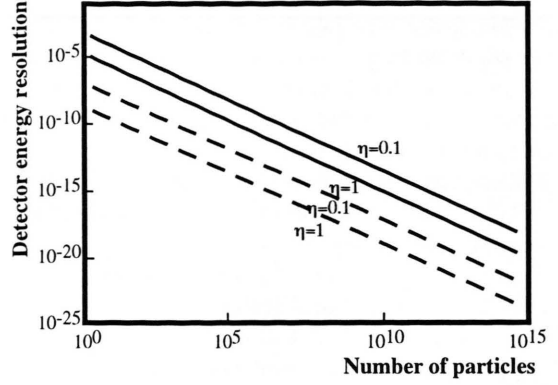

Fig. 2. Needed energy resolution $\Delta E/E_p$ as a function of the number of foil nucleons, for two extremes of the feasibly used light spectrum: x-ray and red (dashed). E_p is the photon energy while $\Delta E = \hbar\omega$. As an example, two η values are given, where η is the Lamb-Dicke parameter equal to the ground state size divided by the light wave length λ, i.e. $(4\pi/\lambda)\sqrt{\hbar/2m\omega}$.

the result of a parity measurement on the outgoing photon.

Taking into account all the above statements, the density matrix ρ_k after the mirror-photon interaction for any initial pure state $|n\rangle$ has the form

$$
\rho_n = |\Omega_f^n\rangle\langle\Omega_f^n| = |\alpha_0|^2|S_{-n}\rangle|0\rangle\langle0|\langle S_{-n}| \quad (7)
$$

$$
+ |\alpha_2|^2|S_{-n+2}\rangle|2\rangle\langle2|\langle S_{-n+2}| + ...,
$$

where

$$
|\alpha_m|^2 = P_{n\to m(\text{even})} = |\langle n|\cos(k^\Delta x)|m\rangle|^2. \quad (8)
$$

The partial trace now simply gives

$$
G = \text{Tr}_m(\rho_n) = \langle0|\rho_n|0\rangle + \langle1|\rho_n|1\rangle + \langle2|\rho_n|2\rangle + ...
$$

$$
= |\alpha_0|^2|S_{-n}\rangle\langle S_{-n}| + |\alpha_2|^2|S_{-n+2}\rangle\langle S_{-n+2}| + ... \quad (9)
$$

As only the even positions on the diagonal of the photon density matrix have non zero elements, it is clear that $\hat{P}G = G$. Hence from (4) we find

$$
\langle\hat{P}_p\rangle = \frac{\sum l_n(\text{Tr}(\hat{P}(G)))}{\sum l_n \text{Tr}(G)} = +1 \quad (10)
$$

Let us close this short letter by looking at the needed energy detection resolution. In Fig. 2 we plot the needed resolution as a function of the mass of

the foil. Obviously, if one wants to maintain a reasonable ground state size (see implications in [4]), as the oscillating mass is increased the frequency has to be decreased and hence the energy level spacing becomes smaller. Consequently, the needed energy detection resolution becomes more stringent.

We see that for the currently available resolutions of 10^{-13}, one can already perform the experiment with a 10^9 particle mirror.

To conclude, we have shown that as long as the energy resolution of our detectors allows us to ignore all odd excitations or relaxations of the mirror, all out going photons should be symmetric and consequently click only in detector D1. This allows us to be sensitive to symmetry breaking events originating from decoherence processes, even if initially the mirror was in a thermal state.

Acknowledgement

Appendix A: Energy Basis Parity Operator

The parity operator in the descrete position space (e. g. the 2 dimensional double slit experiment) is well known and is presented in the introduction. In infinite space, the parity operator may be written as

$$\hat{P}_x = \int | - x'\rangle\langle x'| \, \mathrm{d}\, x' \tag{A1}$$

with $|x\rangle$ being the position eigen state at point x.

As the energy eigen state $|n\rangle$ may be written as $\int \psi_n(x)|x\rangle\mathrm{d}\,x$, where $\psi_n(x) = \langle n|x\rangle$ is the wave function of $|n\rangle$, we have

$$\hat{P}_x|n\rangle = \int | - x'\rangle\langle x'||n\rangle \, \mathrm{d}\, x'$$

$$= \int | - x'\rangle\psi_n(x') \, \mathrm{d}\, x'$$

$$= \int |x'\rangle\psi_n(-x') \, \mathrm{d}\, x'$$

$$= \int (-1)^n \psi_n(x')|x'\rangle \, \mathrm{d}\, x'$$

$$= (-1)^n|n\rangle. \tag{A2}$$

Namely, the eigen values of the parity operator applied on the energy eigen modes $|n\rangle$ are $\lambda_n = (-1)^n$, which means that in this basis, the operator takes a diagonal form with its diagonal matrix elements equal to λ_n, which is what we wanted to prove.

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